

A numerical analysis on the performance of a pressurized twin power piston gamma-type Stirling engine

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ABSTRACT

In this study, a prototype helium-charged twin-power-piston γ -type Stirling engine has been built, and some of its geometrical and operational parameters have been investigated by a numerical model. Data taken from the prototype engine have been used to correct the values of some factors in the numerical model. The results include volume and temperature variations in the expansion and compression chambers, p - v diagrams, and the effects of regeneration effectiveness, the crank radius of the power piston, the initial pressure of working gas, and the rotation speed on engine's power and efficiency. It has been found that regeneration effectiveness poses the most prominent effect on efficiency, while engine speed is most effective on the engine power within the range of engine speed investigated in this study. This study offers invaluable guides for the design and optimization of γ -type Stirling engines with similar construction.

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1. Introduction

The Stirling engine was patented by Robert Stirling in 1816, and it was a scientifically remarkable power machine. Unfortunately, early Stirling engines were not competitive to IC engines mainly because of their small power-to-weight ratio and slow response to speed and power changes, and were literally ignored for quite a long time during the 19th and 20th centuries. Yet, their advantages of high thermal efficiency, low maintenance requirement, flexibility on energy sources, and safe to operate have attracted significant interest in recent decades due to the world's ever growing energy demand, the depletion of fossil fuel, and the urgency to solve the worsening global warming problem. In particular, their ability to use solar energy has made them one of the most promising solutions to global warming. Now Stirling engines have been the focus of many academic studies, both experimentally and numerically [1–3]. A new industry centered on Stirling engines and their applications has emerged [4]. Due to its high energy conversion efficiency, this 200-year-old invention even found its way to NASA's deep space project [5].

Stirling engines come in many different types; they are commonly classified into three different catalogues according to their configurations, namely α , β , and γ types. Among these configurations, the β -type Stirling engines normally run at a high temperature difference (HTD) between the hot and cold ends of the engine, making them very efficient. However, burning fossil fuel

or a complex solar energy collecting system is needed to sustain the high temperature at the hot end of the engine. The high temperature also gives rise to higher thermal radiation losses, higher thermal stress, and the requirement on expensive high-temperature resisting materials. The γ -type engines, on the other hand, normally run on a low or moderate temperature difference, enabling them to be far more flexible on energy sources (biomass or waste heat) or requiring much simpler solar energy collecting techniques. In addition, the material requirement of such engines is less demanding due to the lower temperature at the hot end of the engine; and they can be built cheaply, which is ideal for domestic applications. However, the major disadvantage of the γ -type engines is that they tend to be less efficient and less powerful than the β -type engines mainly because of low temperature difference.

The studies on γ -type engines are comparatively less than those on the β -type engines in open literature. Cinar and Karabalt [6] designed and manufactured a small gamma-type Stirling engine with 276 cc swept volume. The maximum power output of the engine was 128.3 W when charged with 4 bar helium and heated to 1273 K at the hot end. Their work demonstrated that when run on HTD, a gamma-type Stirling engine is also capable of producing respectable power output. Parlak et al. [7] reported a thermodynamic analysis of a gamma-type Stirling engine using a quasi-steady flow model which gave more precise results for the pressure profile than previous models. They concluded that a thermal efficiency of 25% is achievable for a gamma-type Stirling engine charged with nitrogen to 6.5 bar and a hot end temperature of 873 K. Kongtragool and Wongwises [8] built and test a twin power piston and a four power piston γ -type LTD Stirling engines

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Nomenclature

A_{t1}	wall area inside the expansion chamber	W	work (J)
A_{t2}	wall area inside the compression chamber	$\dot{W}$	power (W)
c_p	constant pressure specific heat ($\text{kJ kg}^{-1} \text{K}^{-1}$)	Y	displacement (m)
c_v	constant volume specific heat ($\text{kJ kg}^{-1} \text{K}^{-1}$)		
H	convective heat transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$)	Greeks	
h	enthalpy per unit mass (kJ kg^{-1})	α	thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)
k	thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)	β	rotation angle of displacer journal on the crankshaft
l_1	length of the power piston linkage bar (m)	ε	regeneration effectiveness
l_2	length of the power piston connection rod (m)	ρ	density (kg m^{-3})
l_3	length of the displacer linkage bar (m)	η	engine efficiency
l_4	length of the displacer connection rod (m)	θ	rotation angle of power piston journal on the crankshaft
l_d	height of displacer (m)	θ_p	phase angle between displacer and power piston
m	mass (kg)	μ	fluid dynamic viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
P	pressure (N m^{-2})	ω	engine speed (rad s^{-1})
$\dot{Q}$	heat transfer rate (W)		
R	gas constant ($\text{kJ kg}^{-1} \text{K}^{-1}$)	Superscripts	
R_1	outer radius of the power piston (m)	n	time-step number
R_2	inner radius of the displacer cylinder (m)		
R_d	outer radius of displacer (m)	Subscripts	
R_{t1}	thermal resistance of the expansion chamber (K W^{-1})	c	compression chamber
R_{t2}	thermal resistance of the compression chamber (K W^{-1})	d	displacer
r_1	crank radius of the power piston (m)	e	expansion chamber
r_2	crank radius of the displacer (m)	f	end of a cycle
T	temperature (K)	H	hot end
t	time (s)	i	flow direction from regenerative channel to compression chamber
t_w	thickness of displacer cylinder (m)	j	flow direction from regenerative channel to expansion chamber
U	internal energy (kJ)	L	cold end
u	internal energy per unit mass (kJ kg^{-1})	p	power piston
V	volume (m^3)		
v	velocity (m s^{-1})		

powered by simulated solar energy. The two engines' performance were tested with air at atmospheric pressure; they yielded maximum power and thermal efficiency of 11.8 W and 0.494%, respectively for the twin power piston engine and 32.7 W and 0.809%, respectively for the four power piston engine. Although both engines' performance is not impressive, the author indicated that the engines can be improved by increasing the precision of engine parts and heat source efficiency and adopting helium or hydrogen as working fluid. The underwhelming performance of the engines could be further attributed to the fact that they were not optimized during the design stage. Karabulut et al. [9] pointed out that many parameters such as thermal and physical properties of working gas, temperature difference between hot and cold ends, charge pressure of working gas, and regenerator effectiveness affect the performance of a Stirling engine; and increasing or decreasing the values of these parameters often not only bring improvement on one aspect but also introduce negative impact on another. For example, increasing the heat transfer coefficient of the heater by adding more flow channels would also result in more flow losses and an increase on dead volume. These contradicting effects on engine performance highlight the need to perform parameter optimization during the design stage. Cheng and Yu [10] argued that the design process of a new Stirling engine can be time consuming and expensive because the determination of geometrical parameters of the engine is merely based on the experience of the designers or insufficient experimental information. Therefore they developed a numerical model and applied it on a β -type rhombic-drive Stirling engine. The model took non-isothermal effects, the effectiveness of the regenerative channel, and the thermal resistance of the heating head into account and was proved to be capable of

identifying the most crucial geometrical parameters affecting the engine's performance. However, many losses in real engines remain unaccounted for in this model, hence its accuracy needs to be further verified by experimental data from real engines.

Timoumi et al. [11] indicated that the actual Stirling engine efficiency remains very low compared to the high theoretical efficiency. Real Stirling engines involve very complex compressible fluid dynamics, thermodynamics, and heat transfer phenomena, and an accurate description and understanding of these highly transient phenomena and the engine losses is necessary to optimize engine's design parameters. Several engine losses have been identified by Walker [12] as conduction losses in the heat exchangers, dissipation by pressure drop, shuttle and gas spring hysteresis losses, and according to Timoumi et al. [11], these losses are not usually accounted for in the published work due to their complexity. Costea et al. [13] also indicated that irreversibilities in the thermodynamic cycle have significant importance in predicting the performance of Stirling engines, and analyses based on conventional entropy or exergy techniques do not relate the irreversibilities to the physical phenomena that cause them. Therefore, the objective of this study is to modify Cheng and Yu's model to include additional engine losses and then apply it on a charged twin power piston γ -type Stirling engine to study some critical geometrical and operational parameters to the engine's performance.

2. Mathematical model

We design and built a prototype air- or helium-charged, twin-power-piston γ -type Stirling engine as illustrated in Fig. 1. In a charged Stirling engine, the entire engine, including the crank

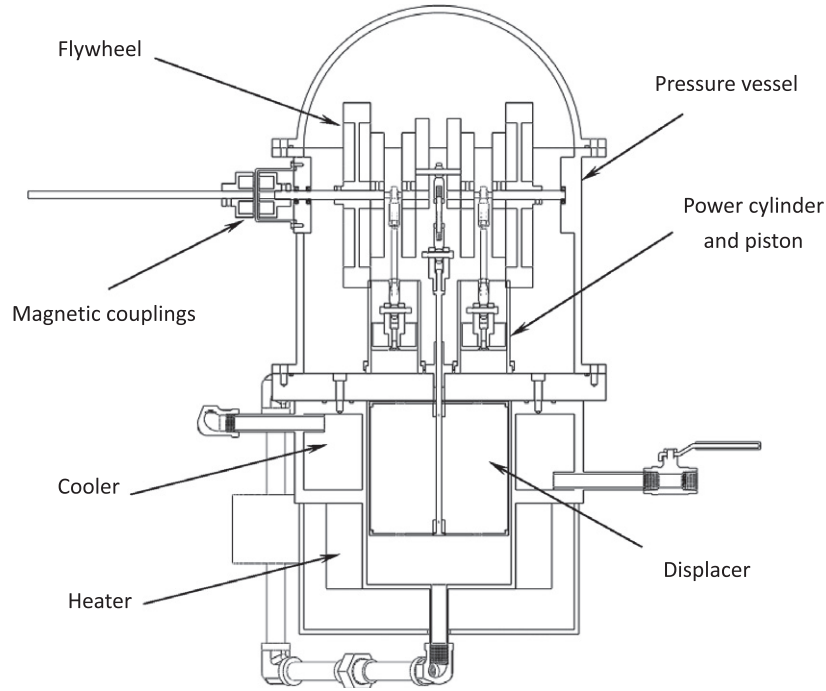


Fig. 1. The design of the current γ -type Stirling engine.

system, is sealed inside a pressure vessel; therefore it is normally equipped with a built-in alternator to convert mechanical energy to electric energy, which can be readily drawn outside the pressure vessel via electric cables. If the engine is used to drive mechanical devices such as fans, pumps, and compressors, an electric motor is needed to convert electric energy back to mechanical energy. This practice introduces inevitable energy conversion losses. For example, even the efficiency of both alternator and electric motor reaches 80%; the two stages of energy conversion still result in almost 40% of total energy loss. To avoid such substantial energy loss, a pair of magnetic couplings is employed here to transmit engine's power through the wall of the pressure vessel to an external power shaft. The rotation of magnetic couplings would produce some eddy current on the metal casing; however, this energy loss is much lower than the energy conversion losses. This feature allows the direct use of the engine's mechanical energy to drive mechanical devices. A schematic of the engine's geometrical parameters is given in Fig. 2, where the engine has been divided into three control volumes: the expansion chamber, the regenerative channel, and the compression chamber. In this configuration, the walls of the expansion and compression chambers also serve as the heater and the cooler, respectively; therefore, there is no external heating or cooling channels. The heater and the cooler are respectively constructed with electric heating coils and fluid jacket surrounding the walls of expansion or compression chambers, thus the heating or cooling areas include not just the top or bottom plates but also parts of the side walls of the displacer cylinder. According to Fig. 2, the displacements of the piston and displacer, $Y_p(t)$ and $Y_d(t)$ are:

$$Y_p(t) = -r_1 \sin \theta + \sqrt{l_1^2 - r_1^2 \cos^2 \theta} + l_2 \quad (1)$$

$$Y_d(t) = -r_2 \sin \beta + \sqrt{l_3^2 - r_2^2 \cos^2 \beta} + l_4 + l_d \quad (2)$$

where $\beta = \theta - \theta_p$, θ_p is the phase angle between the piston and displacer. With the determination of $Y_p(t)$ and $Y_d(t)$, the volumes of the expansion and compression chambers can be expressed as:

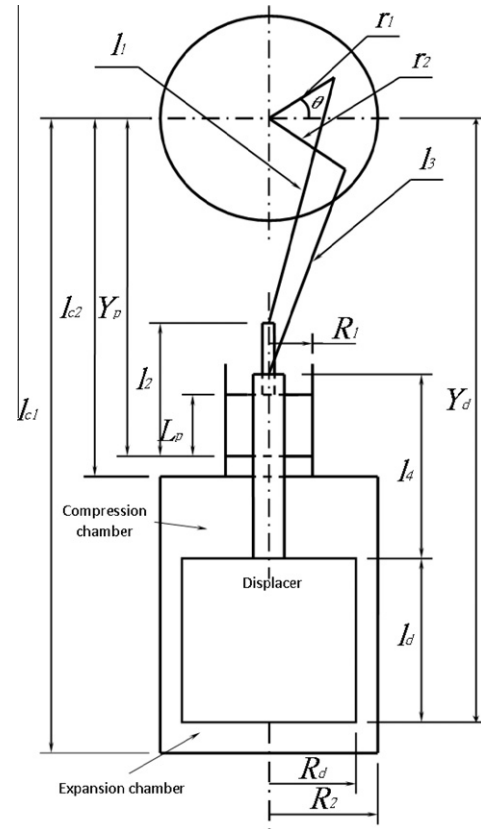


Fig. 2. Definition of engine geometrical parameters.

$$V_e(t) = (l_{c1} - Y_d) \pi R_2^2 \quad (3)$$

$$V_c(t) = (Y_d - l_d - l_{c2}) \pi R_2^2 + 2(l_{c2} - Y_p) \pi R_1^2 \quad (4)$$

where the factor of 2 multiplied to the second term of Eq. (4) accounts for the twin power piston configuration. The velocities of piston and displacer are then calculated by respectively differentiating Eqs. (1) and (2):

$$v_p(t) = -r_1\omega \cos \theta + \frac{r_1^2\omega \cos \theta \sin \theta}{\sqrt{l_1^2 - r_1^2 \cos^2 \theta}} \quad (5)$$

$$v_d(t) = -r_2\omega \cos \beta + \frac{r_2^2\omega \cos \beta \sin \beta}{\sqrt{l_2^2 - r_2^2 \cos^2 \beta}} \quad (6)$$

By using ideal-gas equation of state, the pressure in the expansion and compression chambers are:

$$P_e = \frac{m_e RT_e}{V_e}, \quad P_c = \frac{m_c RT_c}{V_c} \quad (7)$$

Then the energy conservation law is applied respectively to the expansion and compression chambers and regenerative channel; this read:

$$\frac{dU_e}{dt} = \dot{Q}_{in,e} - \dot{Q}_{loss} - \dot{W}_{out,e} - \dot{m} \left(h + \frac{v^2}{2} \right)_e \quad (8)$$

$$-\dot{Q}_{in,r} = \dot{m}(h_{in} - h_{out})_r - \frac{\dot{m}}{2} (v_{in}^2 - v_{out}^2)_r \quad (9)$$

$$\frac{dU_c}{dt} = \dot{Q}_{in,c} + \dot{Q}_{loss} - \dot{W}_{out,c} + \dot{m} \left(h + \frac{v^2}{2} \right)_c \quad (10)$$

where $\dot{Q}_{loss}$ represents the heat losses from the expansion chamber to the compression chamber. According to Timoumi et al. [11], the dominant heat losses are internal conduction heat loss and shuttle heat loss; hence the term $\dot{Q}_{loss}$ is approximated by:

$$\dot{Q}_{loss} \cong \dot{Q}_{cond} + \dot{Q}_{shlt} \quad (11)$$

The internal conduction heat loss is the energy loss due to internal conduction between the hot and cold parts of the engine. In the current configuration, this loss reads:

$$\dot{Q}_{cond} = k_s \frac{2\pi R_2 t_w (T_H - T_L)}{l_{cond}} \quad (12)$$

where t and l_{cond} are the thickness of the displacer cylinder and the distance between the hot and cold ends of the engine, respectively. The shuttle heat loss is rooted in the reciprocating motion of the displacer, which absorbs a quantity of heat from the hot end and restores it at the cold end. Chang and Baik [14] derived a simple expression to account for the shuttle heat loss as:

$$\dot{Q}_{shlt} = \frac{\frac{T_H - T_L}{l_d} \frac{2r_2\pi R_d}{8} \left(\frac{2}{k_s} \sqrt{\frac{\alpha_f}{2\omega}} + \frac{\delta}{k_s} \right)}{\left(\frac{2}{k_s} \sqrt{\frac{\alpha_f}{2\omega}} \right)^2 + \left(\frac{2}{k_s} \sqrt{\frac{\alpha_f}{2\omega}} + \frac{\delta}{k_s} \right)^2} \quad (13)$$

where α_f is the thermal diffusivity of the working gas, and k_s the thermal conductivity of solid material. The above equation is simplified from Chang and Baik's original equation by assuming the displacer cylinder wall and the displacer are constructed by the same solid material. In Eqs. (8)–(10), we assume that the mass flow out of the expansion chamber is positive. Since the working fluid flows in and out of the expansion and compression chambers repeatedly via passing the regenerative channel, a convection scheme is needed to determine the fluid properties flowing in and out of each control volume. Here, a simple upwind scheme is adopted; that is:

$$\dot{m} > 0, \begin{cases} h = h_e, v = v_e; \text{expansion} \\ h_{in} = h_e, h_{out} = h_i, v_{in} = v_e, v_{out} = v_i; \text{regenerative} \\ h = h_i, v = v_i; \text{compression} \end{cases} \quad (14)$$

$$\dot{m} < 0, \begin{cases} h = h_j, v = v_j; \text{expansion} \\ h_{in} = h_c, h_{out} = h_j, v_{in} = v_c, v_{out} = v_j; \text{regenerative} \\ h = h_c, v = v_c; \text{compression} \end{cases} \quad (15)$$

where subscripts i and j represent the flow properties from the regenerative channel to the compression chamber and the flow from the regenerative channel to the expansion chamber, respectively. For flow passing the regenerative channel, the temperatures T_i and T_j are approximated by:

$$\begin{aligned} T_i &= T_e + \varepsilon(T_c - T_e), \quad \dot{m} > 0 \\ T_j &= T_c + \varepsilon(T_e - T_c), \quad \dot{m} < 0 \end{aligned} \quad (16)$$

where ε is the effectiveness of the regenerative channel which poses decisive impact on the performance of the Stirling engine. The internal energy and enthalpy of the working fluid can be calculated by:

$$u = c_v T, \quad h = c_p T \quad (17)$$

The heat transfer rates in the expansion and compression chambers are approximated by:

$$\dot{Q}_{in,e} = \frac{T_H - T_e}{R_{t1}}, \quad \dot{Q}_{in,c} = \frac{T_L - T_c}{R_{t2}} \quad (18)$$

where T_H , R_{t1} and T_L , R_{t2} are the temperatures and thermal resistances respectively at the walls of the expansion and compression chambers. In Cheng and Yu [10], the values of R_{t1} and R_{t2} are fixed. In reality, as the heat transfer areas of the expansion and compression chambers vary with time, these values should also vary with time. The heat transfer areas of the expansion and compression chambers A_{t1} and A_{t2} can be written as:

$$\begin{aligned} A_{t1} &= \pi R_2^2 + 2\pi R_2(l_{c1} - Y_d), \\ A_{t2} &= (\pi R_2^2 - 2\pi R_1^2) + 2\pi R_2(Y_d - l_{c2} - l_d) \end{aligned} \quad (19)$$

and R_{t1} and R_{t2} become:

$$R_{t1} = \frac{1}{HA_{t1}}, \quad R_{t2} = \frac{1}{HA_{t2}} \quad (20)$$

where H is the convective heat transfer coefficient on the walls of expansion and compression chambers. Karabulut et al. [15] suggested the values of H for air and helium in their engine are $447 \text{ W m}^{-2} \text{ K}^{-1}$ and $2392 \text{ W m}^{-2} \text{ K}^{-1}$, respectively. The value for helium will be tested in the current study. The power output is calculated by:

$$\dot{W}_{out,e} = \frac{P_e(V_e^{n+1} - V_e^n)}{\Delta t}, \quad \dot{W}_{out,c} = \frac{P_c(V_c^{n+1} - V_c^n)}{\Delta t} \quad (21)$$

where superscript n represents the marching time step number. Substituting Eqs. (12)–(15) into Eqs. (8) and (10), the temperatures in the expansion and compression chambers can be obtained:

$$\begin{aligned} T_e^{n+1} &= \left(1 - \frac{\Delta t}{\dot{m} c_p R_{t1}} \right) T_e^n \\ &+ \frac{\Delta t}{\dot{m} c_p} \left[\frac{T_H}{R_{t1}} - \frac{P_e(V_e^{n+1} - V_e^n)}{\Delta t} - \dot{m} \left(h_e + \frac{v_e^2}{2} \right) - \dot{Q}_{loss} \right] \end{aligned} \quad (22)$$

for $\dot{m} > 0$, or

$$T_e^{n+1} = \left(1 - \frac{\Delta t}{\bar{m}_e c_v R_{t1}}\right) T_e^n + \frac{\Delta t}{\bar{m}_e c_v} \left[\frac{T_H}{R_{t1}} - \frac{\bar{P}_e (V_e^{n+1} - V_e^n)}{\Delta t} - \dot{m} \left(h_j + \frac{v_j^2}{2} \right) - \dot{Q}_{loss} \right] \quad (23)$$

for $\dot{m} < 0$; and

$$T_c^{n+1} = \left(1 - \frac{\Delta t}{\bar{m}_c c_v R_{t2}}\right) T_c^n + \frac{\Delta t}{\bar{m}_c c_v} \left[\frac{T_L}{R_{t2}} - \frac{\bar{P}_c (V_c^{n+1} - V_c^n)}{\Delta t} + \dot{m} \left(h_i + \frac{v_i^2}{2} \right) + \dot{Q}_{loss} \right] \quad (24)$$

for $\dot{m} > 0$, or

$$T_c^{n+1} = \left(1 - \frac{\Delta t}{\bar{m}_c c_v R_{t2}}\right) T_c^n + \frac{\Delta t}{\bar{m}_c c_v} \left[\frac{T_L}{R_{t2}} - \frac{\bar{P}_c (V_c^{n+1} - V_c^n)}{\Delta t} + \dot{m} \left(h_c + \frac{v_c^2}{2} \right) + \dot{Q}_{loss} \right] \quad (25)$$

for $\dot{m} < 0$, where the bar over a quantity represents the averaged value of that quantity between time steps n and $n+1$:

$$\bar{\phi} = \frac{\phi^{n+1} - \phi^n}{2} \quad (26)$$

In Eqs. (22)–(25), the wall temperatures of the expansion and compression chambers are assumed to be T_H and T_L , respectively. In reality, however, T_H or T_L is maintained at the external walls of the expansion or compression chambers, and heat is transferred into or out of the expansion or compression chambers from the inner walls of both chambers via mechanism of conjugate heat transfer. In this case, the inner wall temperature of the expansion chamber is lower than T_H , and that of the compression chamber is higher than T_L . To take the mechanism of conjugate heat transfer into account, the wall temperatures of the expansion and compression chambers T_H and T_L should be respectively modified as:

$$T_{WH} = \frac{\frac{k_s}{t_w} T_H + H T_e}{\frac{k_s}{t_w} + H}, \quad T_{WL} = \frac{\frac{k_s}{t_w} T_L + H T_c}{\frac{k_s}{t_w} + H} \quad (27)$$

In numerical model, the regenerator is assumed at the annular between the displacer piston and cylinder. The mass flow rate $\dot{m}$ is calculated by solving the viscous velocity profile in the annular regenerative channel and integrating the mass flow rate across the channel; this yields:

$$\dot{m} = -\frac{\rho \pi}{8 \mu} \left(\frac{P_e - P_c}{l_d} \right) \left[R_2^2 - R_d^2 - \frac{(R_2^2 - R_d^2)^2}{\ln(R_2/R_d)} \right] - \frac{2 \pi \rho v_d}{\ln(R_2/R_d)} \left[\frac{1}{2} (R_2^2 \ln R_2 - R_d^2 \ln R_d) - \frac{1}{4} (R_2^2 - R_d^2) \right] + \frac{\pi \rho v_d}{\ln(R_2/R_d)} \ln R_2 (R_2^2 - R_d^2) \quad (28)$$

In the real engine, the regenerator includes not only the annular but also the space inside the displacer piston where very thin copper wires have been put inside to serve as the regenerative material. This is done by drilling a number of small holes on the top and bottom plates of the displacer piston, allowing working gas to pass through the interior of the displacer piston. Hence, the approximation of regeneration channel adopted in the model might be a source of error. The network output per cycle is calculated by the following integration:

$$W_{out} = \int_0^{t_f} P_e dV_e + \int_0^{t_f} P_c dV_c - W_f \quad (29)$$

where t_f represents the time at the end of one cycle, and W_f accounts for frictional losses from all moving parts of the engine.

The magnitude of frictional losses depends on many factors in manufacturing the engine such as the accuracy of machining, clearance between the piston and cylinder, and the alignment of engine parts, especially the crank assembly. Misalignment of crank assembly would inadequately compress shaft bearings and cause a dramatic rise of bearing friction. The friction loss magnitude cannot be predicted; however, it is reasonable to assume that friction loss is constant in every engine cycle, and the power lost to friction $\dot{W}_f$ can be approximated by:

$$\dot{W}_f = \frac{\omega}{2\pi} W_{fc} \quad (30)$$

Finally, the thermal efficiency of the engine can be obtained by:

$$\eta = \frac{W_{out}}{\int_0^{t_f} \dot{Q}_{in,e} dt} \quad (31)$$

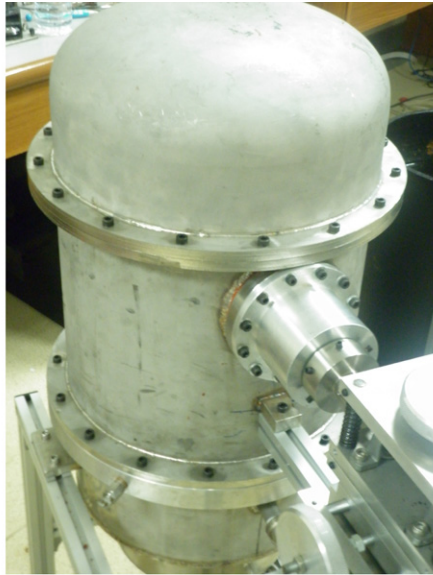
3. Results and discussion

The values of the geometrical parameters of the baseline Stirling engine are given in Table 1. The whole engine, including the crank system and fly wheels, is housed inside a well-sealed 3-mm thick stainless steel pressure vessel, allowing the engine to be charged up to 10 bars with air or helium. Since the current engine has two power pistons and one displacer piston, the crank shaft is consisted of six rectangular sections and three connection rod pins, all joined by welding to provide superior strength. Extreme care was taken to minimize misalignment of these pieces to reduce inadequate compression on shaft bearings. The body of the power piston was made of aluminum, and that of the power cylinder was of brass. The clearance between the power piston and cylinder is 0.02 mm, and the contact surfaces of both were polished to super-finish quality. The displacer piston was constructed using two 2-mm thick circular aluminum top and bottom plates covered with 0.1-mm stainless steel sheet to form the lateral skin. The displacer cylinder was made of steel pipe with inner diameter of 160 mm and wall thickness of 3 mm. The cooler is a water jacket welded around the upper part of the displacer cylinder, through which fresh cooling water is circulating to maintain low temperature, and the heater is a thermostat controlled electric heater, capable of heating up to 873 K, wrapped on the lower part of the displacer cylinder. Fig. 3a shows the entire engine, where the pressure vessel and the external magnet of the magnet couplings can be seen. In Fig. 3b, the interior of the pressure vessel is shown, including the crank system, twin power cylinders, and the flywheels. A torque transducer, KISTLER 4520A100 with accuracy of 0.1 N-m, was installed between the engine's power shaft and the loading to measure the engine's torque. A photo tachometer, TECPEL UT371 with accuracy of 0.01 RPM, was employed to measure

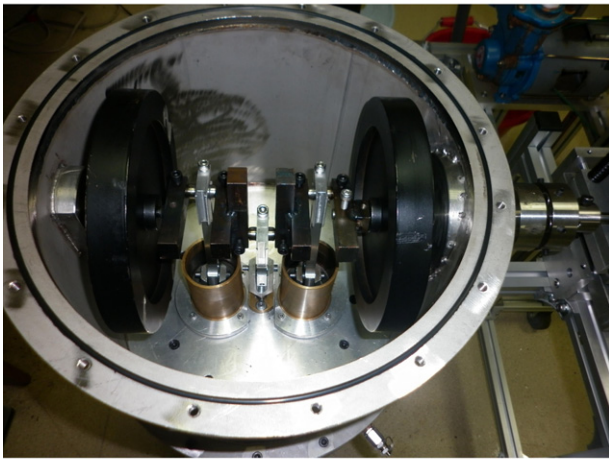
Table 1

The values of the geometrical parameters of the baseline Stirling engine.

R_1 (m)	0.0256
R_2 (m)	0.079
R_d (m)	0.077
r_1 (m)	0.025
r_2 (m)	0.025
l_1 (m)	0.13
l_2 (m)	0.045
l_3 (m)	0.095
l_4 (m)	0.155
l_d (m)	0.148
l_{c1} (m)	0.425
l_{c2} (m)	0.221
t (m)	0.003
θ_p (°)	90



(a)



(b)

Fig. 3. The prototype Stirling engine, (a) the pressure vessel, and (b) the interior of the pressure vessel, including twin power cylinders, crank system, and flywheels.

the engine's rotation speed. The measurements from the torque transducer and the tachometer allow the engine's power to be calculated. Since using a real Stirling engine to conduct a parametric study is not practical, in this study, the prototype Stirling engine only serves the purpose of providing data to calibrate the numerical model.

The numerical code is first validated using the results in Cheng and Yu [10], where a β -type rhombic-drive Stirling engine was simulated, and results in terms of volume, temperature, pressure, and heat transfer variations in Stirling cycles were well documented for a base-line engine. Details of the geometric and operation parameters of the base-line case can be found in [10] and will not be repeated here. The calculation in [10] did not take heat-transfer and friction losses, and effect of conjugate heat transfer into account; therefore, these losses are set to zero, and conjugate heat transfer effect is ignored in the current code for code-validation purpose. As shown in Fig. 4, the comparison of p - v diagram of the expansion and compression chambers indicates good agreement between the results obtained by the current code and those in [10]. The calculated engine power of 16.7 W is also very close to the value of 16.75 W reported in [10]. This exercise has verified the correctness of the current numerical code.

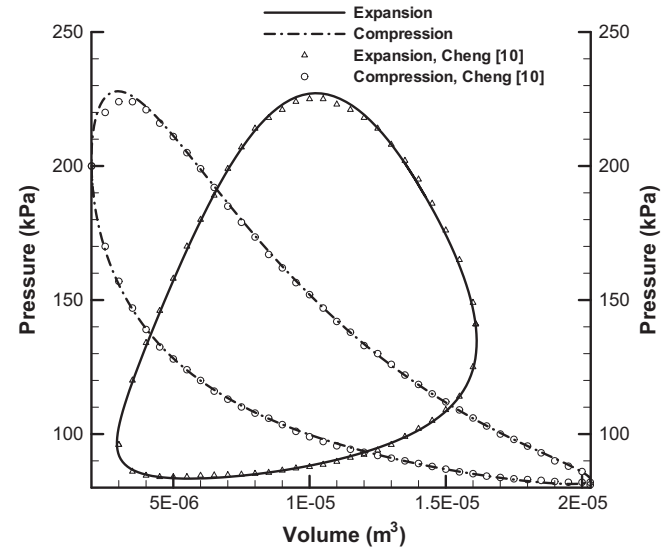


Fig. 4. Comparison of expansion and compression chambers p - v diagrams of the current code and Cheng and Yu [10].

Before the numerical simulation of the gamma-Stirling engine, the value of W_{fc} in Eq. (29) needs to be determined. Since this value is highly dependent on the manufacture quality of an engine, it can only be determined by measurement. A simple method is used to estimate this quantity. First, the prototype engine was run with air under atmospheric pressure, and T_H and T_L were fixed at 473 K and 300 K, respectively. In these conditions, the engine only yielded small power. A small loading was applied, and the measured engine power was 3.6 W at the engine speed of 104 rpm. Then the loading was removed, and the engine speed was increased by 196 rpm to 300 rpm. When the loading was removed, the engine's power was consumed entirely by friction loss. Assuming that engine's power output is the same at both engine speeds, the power output of 3.6 W is equivalent to $\dot{W}_f$, and the increased engine speed of 196 rpm is equivalent to the rotational speed. In this case, the value of W_{fc} is 1.08 J/cycle. This method only gives a rough estimation on W_{fc} because the engine's power output actually depends on engine's speed.

The thermodynamic cycle of the prototype engine under a set of baseline operation conditions is simulated by the code. The baseline operation parameters are as follows: engine speed = 400 rpm, hot end temperature $T_H = 700$ K, cold end temperature $T_L = 300$ K, the working gas is helium charged to 2 bar, convective heat transfer coefficient at hot and cold ends $H = 2396 \text{ W m}^{-2} \text{ K}^{-1}$, and regeneration effectiveness $\varepsilon = 0.3$. Here, the engine speed is typical for a γ -type Stirling engine, and the predefined hot end temperature is reachable by burning biomass or using simple one-axis tracking parabolic trough solar collector. Figs. 5–7 respectively show the volume variations, temperature variations, and p - v diagrams of the expansion and compression chambers. Fig. 5 shows that the variations of volume in the expansion and compression chambers are similar; however, the minimum compression volume is larger than the minimum expansion volume because there are dead volumes in the twin power cylinders. As can be seen in Fig. 6, the variation of temperature in the expansion chamber is about 70 K, and that of the compression chamber is about 50 K. Cheng and Yu [10] reported the simulated maximum expansion chamber temperature is a bit higher than the hot end temperature, and the minimum compression chamber temperature is a bit lower than the cold end temperature. They attributed the phenomena to the high compression ratio of their β -type engine. Such overshooting phenomena are not observed here presumably due to the much

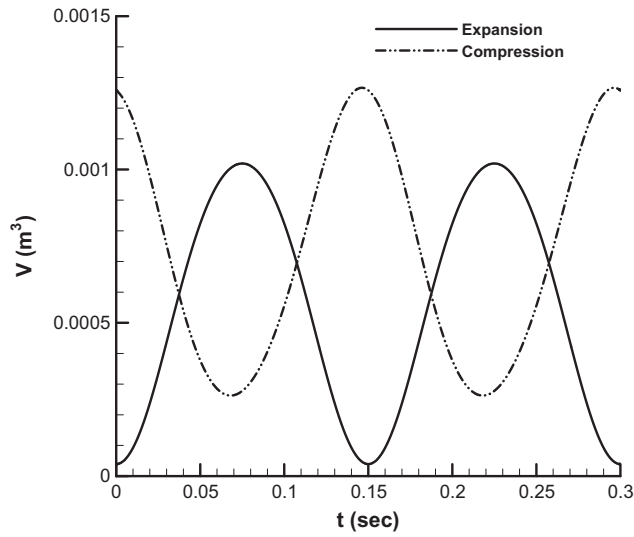


Fig. 5. Volume variations in the expansion and compression chambers under baseline conditions.

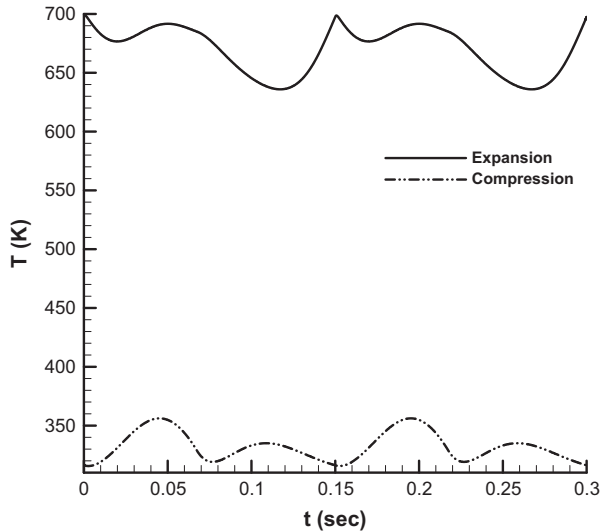


Fig. 6. Temperature variations in the expansion and compression chambers under baseline conditions.

lower compression ratio in the current engine and the introduction of the conjugate heat transfer effect in the numerical model. From Fig. 7, it can be seen that during one engine cycle, the expansion chamber yields positive net work, whereas the compression chamber produces negative net work. The absolute value of the positive net work is larger than that of the negative net work, giving rise to positive work output to the engine's shaft. Therefore, engine's efficiency can be improved by promoting the positive net work in the expansion chamber and reducing the negative net work in the compression chamber. The calculated engine power is 159.77 W for the baseline conditions. This value, however, is significantly overestimated when compared with the measured power of 102 W. The calculated heat transfer rate into the expansion chamber is 2.3 kW, which is close to the input electric power. Some of the reasons contributing to the overestimation of engine power could be that the values of convective heat transfer coefficient H and the regeneration effectiveness ε are too high in the model. By resetting the values of H and ε to $1500 \text{ W m}^{-2} \text{ K}^{-1}$ and 0.2, respectively, the calculated engine power becomes 146.1 W, much

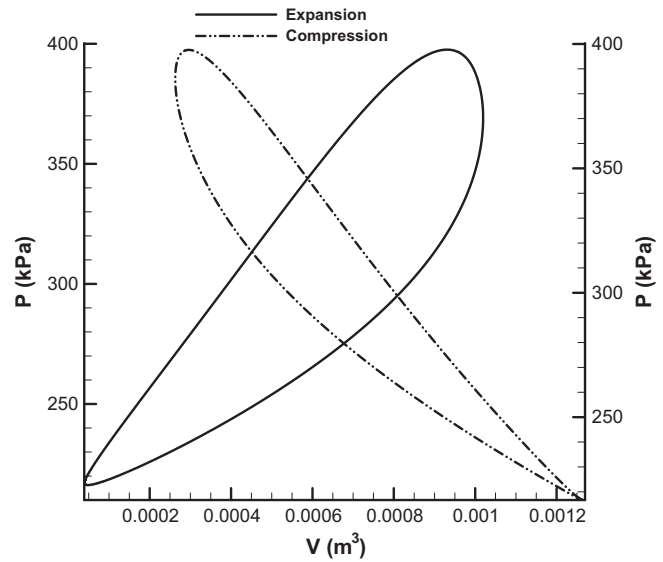


Fig. 7. p - v diagrams in the expansion and compression chambers under baseline conditions.

closer to the measured value. Other reasons responsible for the inaccuracy of the model could be more fundamental, such as the assumption of annular regeneration channel and uniform pressure inside the expansion and compression chamber at any given moment. To take the effects of these factors into account, a 3D CFD simulation using moving grid technique and solving Navier–Stokes equations is needed. Yet, such undertaking is very sophisticated and time consuming and is not appropriate for parametric study. In the following calculations, the smaller H value will be adopted.

Next, the effects of four important geometrical and operational parameters, namely the regeneration effectiveness, the crank radius of the power piston r_1 , the initial pressure of working gas, and the rotation speed, on the engine performance are examined. In this analysis, the working gas is helium, and the hot- and low-end temperatures are fixed at 700 K and 300 K, respectively. The regeneration effectiveness has been well known as one of the most influential factors promoting the engine's power and efficiency. The value of r_1 determines the swept volume of the power pistons, which, according to [6], poses crucial impact on the engine's

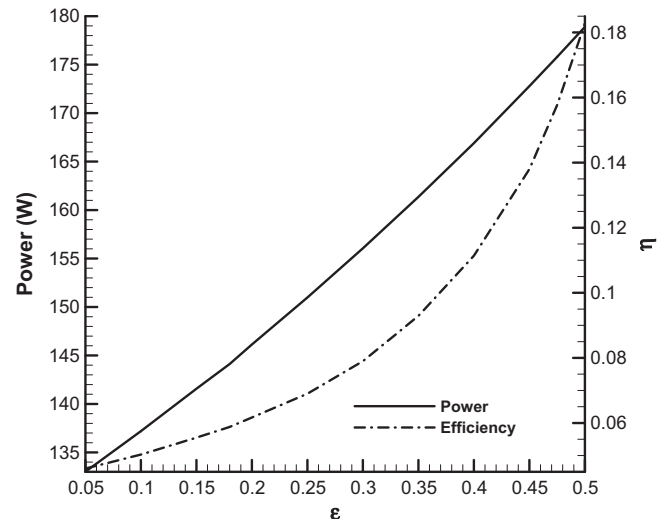


Fig. 8. The effects of regeneration effectiveness on engine's power and efficiency.

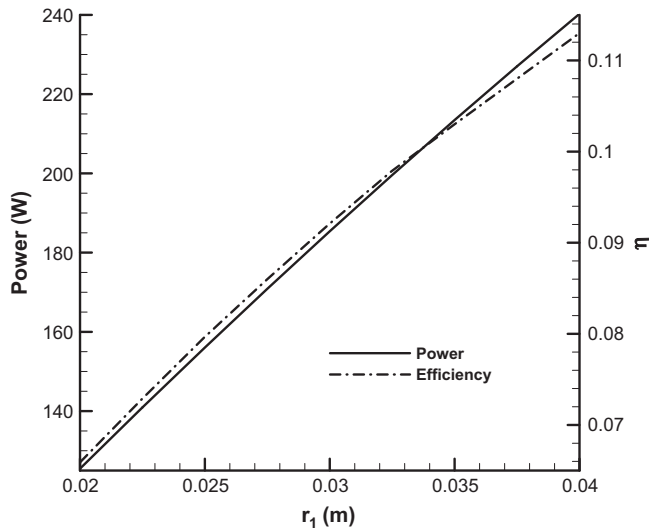


Fig. 9. The effects of power piston crank radius on engine's power and efficiency.

performance. The initial pressure dictates the amount of working gas inside the engine and is also influential to power output. The engine speed is yet another very crucial factor on its power and efficiency. The effects of these parameters are respectively shown in Figs. 8–11. In Fig. 8, the values of r_1 , initial pressure, and engine speed are fixed at 0.025 m, 2 bars, and 400 rpm respectively. The results clearly indicate that both the engine's power and efficiency are proportional to ε . Its effect is more pronounced on the engine's efficiency which increases more than three times when ε increases from 0.05 to 0.5. To investigate the effect of r_1 , the values of ε , initial pressure, and engine speed are fixed at 0.3, 2 bars, and 400 rpm, respectively. It is noticeable in Fig. 9 that both engine's power and efficiency increase almost linearly with the increase of r_1 ; especially the power is increased by a factor of 2 as r_1 changes from 0.02 m to 0.04 m. In terms of investigating the effect of initial pressure, the values of ε , r_1 , and engine speed are fixed at 0.3, 0.025 m and 400 rpm, respectively. As can be seen in Fig. 10, the power and efficiency also increases almost linearly as pressure increases. This parameter is very effective on both power and efficiency. Finally, to examine the effects of engine speed, the values of ε , r_1 , and initial pressure are fixed at 0.3, 0.025 m, and 2 bar,

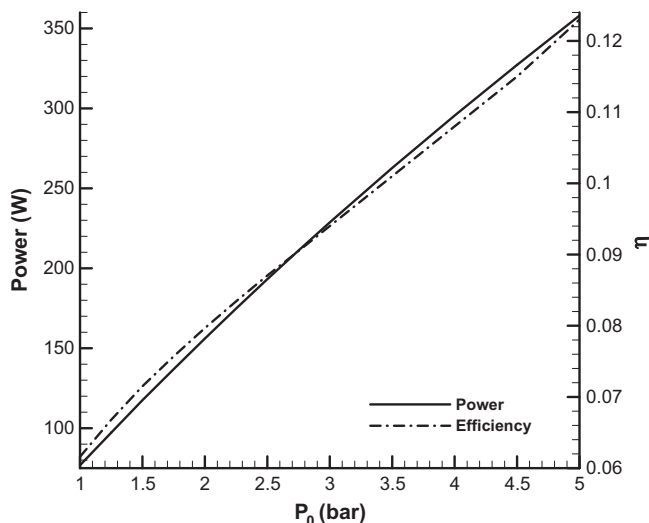


Fig. 10. The effects of initial change pressure on engine's power and efficiency.

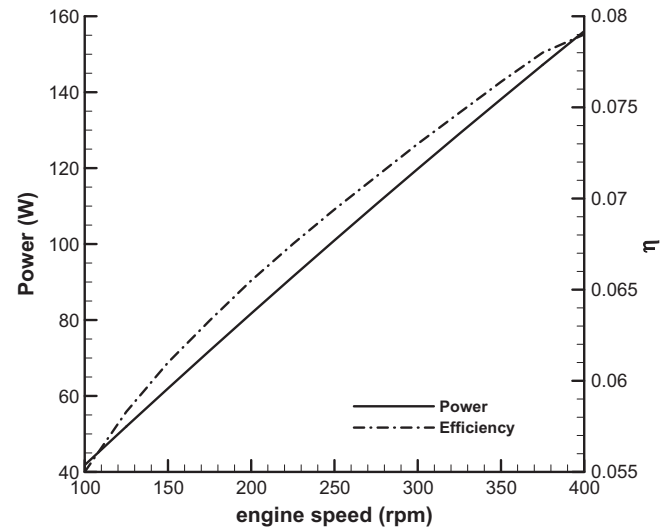


Fig. 11. The effects of engine speed on engine's power and efficiency.

respectively. Fig. 11 indicates that engine speed is of great importance on engine power. As the engine speed increases from 100 rpm to 400 rpm, the power increases from 40 W to 160 W, a leap of four folds. This seems to suggest that engine power can be readily boosted by just increasing engine speed. However, the γ -type engine tends to have a large displacer piston. Particular in the current configuration, the displacer also houses the regeneration material, making it a heavy component. Its mass is between 0.5 and 0.8 kg, depending on the amount of regeneration material put inside. The heavy weight of the displacer limits the engine speed because its reciprocation motion inside the displacer cylinder results in huge change of momentum at high speed. The momentum change acting on the crank shaft causes observable unbalance and vibration as engine speed reaches 400 rpm. Therefore, this type of engine is not engineered to run at high speed, and the engine speed of 400 rpm is more or less the safe top speed of the current γ -type Stirling engine. The impact of engine speed on efficiency is less dramatic as efficiency only increases from 0.55 to 0.79 when engine speed increases from 100 to 400 rpm.

4. Conclusions

In this paper, a parametric study has been conducted on a helium-changed twin-power-piston γ -type Stirling engine. A baseline engine has been constructed to correct some parameters in the numerical model. Four important geometrical and operational parameters have been investigated, including the regeneration effectiveness, the crank radius of the power piston, the initial pressure of working gas, and the rotation speed. It has been found that regeneration effectiveness poses the most prominent effect on efficiency, while increasing engine speed is most effective on the engine power within the range of engine speed investigated. The crank radius of power piston and initial gas pressure, on the other hand, are also very influential on engine power. Their effects on engine efficiency, however, are not as dramatic. This study offers invaluable information on the optimization of the prototype γ -type Stirling engine and the design of other γ -type engines with similar construction.

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